JHEP HYPER VERSION

Received: *June 29, 1999*, Accepted: *October 15, 1999*

Vacuum oscillations of quasi degenerate solar neutrinos*

Riccardo Barbieri
Scuola Normale Superiore and INFN, sezione di Pisa
I-56126 Pisa, Italia
E-mail: barbierr@cibs.sns.it

Graham G. Ross
Department of Physics, University of Oxford
1 Keble Road, Oxford OX1 3NP, UK
E-mail: g.ross@physics.oxford.ac.uk

Alessandro Strumia
Dipartimento di fisica, Università di Pisa and INFN, sezione di Pisa
I-56126 Pisa, Italia
E-mail: astrumia@difi.unipi.it

Abstract: The atmospheric neutrino oscillations and the vacuum oscillation solution of the solar neutrino problem can be consistently described by a doubly or triply degenerate neutrino spectrum as long as the high level of degeneracy required is not spoiled by radiative corrections. We show that this is the case for neutrino mass matrices generated by symmetries. This imposes a strong constraint on the mixing angles and requires the mixing should be close to bi-maximal. We briefly discuss the relevance of our results for the measurability of the neutrino spectrum.

Keywords: Solar and Atmospheric Neutrinos, Neutrino Physics.

*Research supported in part by the EEC under TMR contract ERBFMRX-CT96-0090.

JHEP10(1999)020

Contents

1. Introduction

Observations of atmospheric and solar neutrinos provide very significant indications that neutrinos oscillate between different mass eigenstates m_i [1, 2]. As a result, progress has been made in experimentally determining these masses as well as the related mixing parameters. In what follows we assume just 3 light Majorana neutrinos and we use a standard notation for the leptonic mixing matrix

$$V = R_{23}(\theta_{23})\,\mathrm{diag}(1, e^{i\phi}, 1) R_{13}(\theta_{13}) R_{12}(\theta_{12})\,, \tag{1.1}$$

where $R_{ij}(\theta_{ij})$ represents a rotation by θ_{ij} in the ij plane. Within this framework and with this notation, the present situation can be summarised as follows:

- It is very likely, although still awaiting confirmation, that

$$\Delta m^2_{23} \equiv m^2_{\rm atm} = 10^{-(3\div 2)}\,\mathrm{eV}^2\,, \qquad \sin^2 2\theta_{23} \gtrsim 0.8\,. \tag{1.2}$$

- It is not unlikely that

$$\Delta m^2_{12} \equiv m^2_{\rm sun} \lesssim 10^{-4}\,\mathrm{eV}^2\,, \qquad \sin^2\theta_{13} \lesssim 0.1 \tag{1.3}$$

  with [3]

$$3 \cdot 10^{-6}\,\mathrm{eV}^2 \lesssim m^2_{\rm sun} \lesssim 10^{-5}\,\mathrm{eV}^2\,, \qquad 2 \cdot 10^{-3} < \sin^2 2\theta_{12} < 2 \cdot 10^{-2} \tag{1.4}$$

  or

$$10^{-5}\,\mathrm{eV}^2 \lesssim m^2_{\rm sun} \lesssim 10^{-4}\,\mathrm{eV}^2\,, \qquad 0.6 \lesssim \sin^2 2\theta_{12} < 0.95 \tag{1.5}$$

  or

$$5 \cdot 10^{-11}\,\mathrm{eV}^2 \lesssim m^2_{\rm sun} \lesssim 10^{-9}\,\mathrm{eV}^2\,, \qquad \sin^2 2\theta_{12} \gtrsim 0.6 \tag{1.6}$$

  corresponding respectively to the small angle MSW (SAMSW), large angle MSW (LAMSW) or vacuum oscillation (VO) solutions of the solar neutrino problem.

It is clearly of great importance to confirm or disprove this picture and further constrain the allowed range of the parameters.

Even accepting (1.2), (1.3) and (1.4)–(1.6), which we do hereafter, the neutrino spectrum is not determined. As is well known, three different possibilities exist:

1. "degenerate": $m_1 \approx m_2 \approx m_3 \gtrsim m_{\rm atm}$;
2. "pseudo-Dirac": $m_1 \approx m_2 \approx m_{\rm atm} \gg m_3$;
3. "hierarchical": $m_3 \approx m_{\rm atm} \gg m_2 \approx m_{\rm sun} \gtrsim m_1$.

It is extremely important to know, with a minimum of theoretical bias, which spectrum is realized in nature. However, at the moment we only know that the heaviest neutrino weights less than a few eVs from direct β-decay searches or from astrophysical and cosmological data. Three different areas of experimental developments can have an impact on this issue:[1]

1. The determination of the actual solution of the solar neutrino problem.
2. The neutrinoless double-beta decay searches.
3. The cosmological signals of a neutrino rest mass.

Prior to the discussion of the experimental potential in this area there is, however, one relevant theoretical problem. The VO of solar neutrinos can be consistently described by the 'degenerate' or the 'pseudo-Dirac' spectra only as long as the high level of degeneracy required is not spoiled by radiative corrections [4, 5]. We investigate this question in this paper, concentrating on the form of neutrino mass matrices motivated by symmetries, as previously suggested [6, 7, 8]. These mixing matrices are all characterized by having $\theta_{12} = \pi/4$ and $\theta_{13} = 0$. We will show that these mass matrices have a sufficient degree of stability against radiative corrections to make the VO solution consistent with both doubly and triply degenerate neutrino spectra. In conjunction with the experimental requirement that θ_{23} is approximately $\pi/4$, this leads to the so called bimaximal mixing matrix [9]. Similar conclusions have also been reached in [10, 11]. Finally we will consider the necessary and sometimes sufficient conditions needed to determine the full neutrino spectrum.

[1]We remind that in this paper we are considering the case there are just three light neutrinos. Of course our analysis would require revision if this proves not to be the case.

JHEP10(1999)020

2. Radiative corrections to the neutrino mass textures

We concentrate on neutrino mass matrices, in the charged lepton flavour basis, of the form

$$M_\nu^0 = m\ R_{23}(\theta_{23}) \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & z \end{pmatrix} R_{23}^T(\theta_{23})\,, \tag{2.1}$$

where: i) for the 'degenerate' case, m is the common neutrino mass, $m \gtrsim m_{\rm atm}$, and $z = e^{i\phi}(1+\delta)$ with $2\delta + \delta^2 = (m_{\rm atm}/m)^2$; ii) for the 'pseudo-Dirac' case, $m = m_{\rm atm}$ and $|z|$ is negligibly small.

In both cases the small splitting necessary to describe the VO of solar neutrinos is neglected. It could come from an explicit extra term in (2.1) or it could even be generated by the same radiative correction effects that we are going to discuss. $R_{23}(\theta_{23})$ is the large angle rotation in the 23 sector that accounts for atmospheric neutrino oscillations. As noted above the neutrino mass matrix (2.1) is in the flavour basis, i.e. it is associated with diagonal charged lepton mass matrices. For the 'degenerate' case, texture (2.1) was motivated in [8] on the basis of a spontaneously broken SO(3) flavour symmetry. For the 'pseudo-Dirac' case, $z = 0$, texture (2.1) was introduced in [6, 7] as a consequence of an unbroken abelian symmetry, $L_e - L_\mu - L_\tau$. The very fact that this last symmetry is compatible with the Yukawa couplings (the charged fermion masses) of the Standard Model or of the Minimal Supersymmetric Standard Model makes it clear that radiative corrections will not destabilize (2.1) in either case. The issue is more tricky in the fully 'degenerate' case, with $|z| \approx 1$, since (2.1) is obtained, together with a diagonal charge lepton matrix, only after spontaneous symmetry breaking of the SO(3) symmetry with scalar vacuum expectation values in appropriate directions [8]. Note that (2.1), which must be viewed as an initial condition valid at some scale Λ, can be rewritten as

$$M_\nu^0 = m\ V^* \cdot {\rm diag}(-1, 1, z) \cdot V^\dagger \tag{2.2}$$

with

$$V = R_{23}(\theta_{23}) \cdot R_{12}\left(\frac{\pi}{4}\right).$$

In full generality, up to universal corrections and negligibly small effects of the muon and electron Yukawa couplings, the renormalized neutrino mass at a scale μ below Λ is given in logarithmic approximation by [4, 5]

$$M_\nu = I_\tau \cdot M_\nu^0 \cdot I_\tau\,, \qquad I_\tau = {\rm diag}(1, 1, 1+\epsilon)\,, \tag{2.3}$$

where

$$\epsilon = \frac{g_\tau^2}{(4\pi)^2} \ln\frac{\Lambda}{\mu} \left\{\frac{1}{2}, \frac{-1}{\cos^2\beta}\right\}$$

and $g_\tau = m_\tau/v \approx 0.01$ is the τ Yukawa coupling in the SM. The two factors in parenthesis stand for the SM or for the MSSM contributions respectively, with

$\tan\beta = v_2/v_1$ being the usual parameter related to the ratio of the Higgs vevs. For the purposes of this discussion it is sufficient to take $\mu = M_Z$, ignoring the small corrections due to the different possible thresholds at the electroweak scale. Since $(g_\tau/4\pi)^2 \approx 0.6\ 10^{-6}$, ϵ is significantly larger, for any value of Λ and $\tan\beta$, than the relative splitting needed to account for the VO solution of solar neutrinos,

$$\frac{\Delta m}{m} \leq \frac{m^2_{\rm sun}}{m^2_{\rm atm}} \lesssim 10^{-7}. \tag{2.4}$$

Thus if $\Delta m/m = \mathcal{O}(\epsilon)$ there will be a conflict with the required level of degeneracy of the renormalized neutrino masses in (2.3) [4, 5].

We will demonstrate that for the neutrino mass matrices of the form given in (2.1), due to the underlying symmetries, in fact $\Delta m/m = \mathcal{O}(\epsilon^2)$ and hence the vacuum oscillation solution is quite stable against radiative corrections. It is most convenient to discuss the symmetries in the basis rotated by θ_{23} in which the neutrino mass matrix has the form

$$\tilde{M}^0_\nu = R^T_{23}(\theta_{23}) M^0_\nu R_{23}(\theta_{23}) = m \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & z \end{pmatrix}. \tag{2.5}$$

This is invariant under a U(1) rotation under which the states $1, 2$ and 3 have charges $+1$, -1 and 0 respectively. Now consider the effect of the radiative corrections. If they preserve the U(1) they will leave the zero structure of the mass matrix intact and in turn this leaves one degenerate pair of neutrinos. It is useful to rewrite (2.3) in terms of the matrix $\tilde{I}_\tau$ defined by

$$\tilde{I}_\tau = R^T_{23} I_\tau R_{23} = \mathbb{1} + \epsilon X\ ,$$

where

$$X = \begin{pmatrix} 0 & 0 & 0 \\ 0 & s^2_{23} & c_{23}s_{23} \\ 0 & c_{23}s_{23} & c^2_{23} \end{pmatrix} \tag{2.6}$$

and $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$. In terms of $\tilde{I}_\tau$ the renormalized mass matrix in the same basis is given by

$$\tilde{M}_\nu = \tilde{I}_\tau \cdot \tilde{M}^0_\nu \cdot \tilde{I}_\tau\ . \tag{2.7}$$

Let us consider the order at which the degeneracy of the light neutrinos is lifted. Since $\tilde{I}_\tau$ comes from wave function renormalization, only its diagonal elements are invariant under the U(1) discussed above. Thus U(1) breaking effects arise through the elements X_{23}, X_{32}. But these matrix elements, being off-diagonal, would remove the degeneracy of the eigenvalues at $\mathcal{O}(\epsilon)$ only if $\epsilon \gtrsim |z-1|$, which is not the case, as required by the atmospheric neutrino anomaly.

3. Discussion of the results

We turn now to the quantitative determination of these effects. To do this is useful to rewrite (2.3) in terms of the matrix

$$I'_\tau = V^\dagger I_\tau V \tag{3.1}$$

as

$$M_\nu = V^* \cdot I'^T_\tau M^0_{\text{diag}} I'_\tau \cdot V^\dagger = V^* \cdot M'_\nu \cdot V^\dagger \tag{3.2}$$

where, by explicit calculation,

$$\frac{M'_\nu}{m} = \begin{pmatrix} -1-\epsilon s^2_{23} & 0 & \epsilon'(-1+z)/4 \\ 0 & 1+\epsilon s^2_{23} & \epsilon'(-1-z)/4 \\ \epsilon'(-1+z)/4 & \epsilon'(-1-z)/4 & z(1+2\epsilon c^2_{23}) \end{pmatrix}, \tag{3.3}$$

$\epsilon' \equiv \sqrt{2}\epsilon \sin 2\theta_{23}$. Eqs. (3.2) and (3.3) are the basic expressions for the renormalized neutrino mass matrix in the flavour basis. The renormalization of the 'pseudo-Dirac' spectrum is immediately obtained by setting $z = 0$ in (3.3). This gives

$$M_\nu(z=0) = m\ R_{23}(\theta_{23}) \cdot \begin{pmatrix} 0 & 1+\epsilon s^2_{23} & -\epsilon s_{23} c_{23} \\ 1+\epsilon s^2_{23} & 0 & 0 \\ -\epsilon s_{23} c_{23} & 0 & 0 \end{pmatrix} \cdot R^T_{23}(\theta_{23}), \tag{3.4}$$

which keeps the original form, as anticipated by our symmetry arguments, with (small) renormalizations of the angle θ_{23} and of the overall scale m.

For the degenerate case, $z = e^{i\phi}(1+\delta)$, it is best to work with the hermitean matrix

$$M_\nu M^\dagger_\nu = V^* M'_\nu M'^\dagger_\nu V^T, \tag{3.5}$$

where, from (3.3)

$$M'_\nu M'^\dagger_\nu = m'^2 \begin{pmatrix} 1 & 0 & i\epsilon' e^{-i\phi/2} \sin\frac{\phi}{2} \\ 0 & 1 & -\epsilon' e^{-i\phi/2} \cos\frac{\phi}{2} \\ \text{h.c.} & \text{h.c.} & 1+2\delta \end{pmatrix}, \tag{3.6}$$

with $m' = m(1+\epsilon s^2_{23})$ and up to irrelevant corrections. The eigenvalues of this matrix, i.e. the renormalized squared neutrino masses are

$$\begin{aligned} m_1^2 &= m'^2 \\ m_2^2 &= m'^2(1+\delta-(\delta^2+\epsilon'^2)^{1/2}) \approx m'^2\left(1-\frac{\epsilon'^2}{2\delta}\right) \\ m_3^2 &\approx m'^2(1+2\delta)\,. \end{aligned} \tag{3.7}$$

As anticipated by our symmetry argument the degeneracy between the light states is lifted at $\mathcal{O}(\epsilon^2)$. Note that had we dropped δ in m_2^2 the correction would have been of order ϵ. The inclusion of the atmospheric mass splitting is crucial and explains why our conclusions about the compatibility of a 'degenerate' neutrino spectrum with VO solar oscillations differ from [4, 5].

To the extent (2.1) represents the exact initial condition for M_ν, these renormalized eigenvalues would give[2]

$$\frac{m^2_{\rm sun}}{m^2} = \frac{\epsilon^2}{\delta}\sin^2 2\theta_{23}\,, \qquad \frac{m^2_{\rm atm}}{m^2} = 2\delta\,, \tag{3.8}$$

i.e.

$$m^2_{\rm sun} m^2_{\rm atm} = 2m^4\epsilon^2\ \sin^2 2\theta_{23}\,. \tag{3.9}$$

More generally there could be a splitting of the original unrenormalized eigenvalues, which makes (3.9) a rough estimate of a lower bound on $m^2_{\rm sun}m^2_{\rm atm}$, barring strong accidental cancellations. Numerically, choosing $\Lambda = 10^{5\div 16}\,$GeV[3]

$$\frac{m^2_{\rm sun}m^2_{\rm atm}}{(1\div 20)10^{-11}\,{\rm eV}^4} \gtrsim \left(\frac{m}{\rm eV}\right)^4 \left\{1\left(\frac{2}{\cos^2\beta}\right)^2\right\} \tag{3.10}$$

to be compared with

$$m^2_{\rm sun}m^2_{\rm atm}|_{\rm exp} = 10^{-(11\div 14)}\,{\rm eV}^4\,. \tag{3.11}$$

Eq. (3.10) is our main result and clearly shows that even a threefold degenerate spectrum, with the neutrino mass matrix following from an underlying symmetry, is compatible with the VO solution of the solar neutrino problem. The radiative correction due to the τ Yukawa coupling is actually a candidate for generating the VO $m^2_{\rm sun}$ splitting. If this is the case, i.e. the bound in (3.10) is saturated, a reduction of the uncertainty in the right handed side of (3.11) would allow a rather precise determination of the average neutrino mass m. At present the lower bound for m is given by $m_{\rm atm} = (0.03 \div 0.1)\,$eV. The upper bound follows from (3.10). Note that values of cosmological interest, $\sum_\nu m_\nu \sim$ eV, cannot be safely excluded on the basis of (3.10).

It is of interest to note that $\theta_{12} = \pi/4$ and $\theta_{13} = 0$ have been both necessary to avoid corrections of order ϵ to $m^2_{\rm sun}$ that would drastically change our conclusions (on the contrary the value of θ_{23} does not crucially affect the magnitude of the radiative corrections). At first sight it could appear that the complex 12 rotation necessary to re-diagonalize the RGE-corrected mass matrix (3.6)

$$U_{12}(\phi/2) = {\rm diag}(i,1,1)R_{12}(\phi/2)\,{\rm diag}(-i,1,1)$$

[2]The signs of the mass squared splittings, $m^2_{\rm sun}$ and $m^2_{\rm atm}$, are irrelevant.

[3]For simplicity, we are neglecting the running of the τ Yukawa coupling. In the MSSM with moderate $\tan\beta$ and for $\Lambda \approx 2\ 10^{16}\,$GeV, our approximation is ~ 2 times larger than the exact result.

JHEP10(1999)020

induces a too large renormalization of θ_{12} unless the phase ϕ is very small. This would be a problem for the model in [8], where $\phi \approx \pi/2$. However this complex rotation does not affect $\theta_{12} = \pi/4$, as may be shown by means of the identity

$$U_{12}(\phi/2) R_{12}(\pi/4) = R_{12}(\pi/4)\,\mathrm{diag}(e^{i\phi/2}, e^{-i\phi/2}, 1)\,.$$

Consequently the small RGE effects only induce small RGE corrections, of order ϵ/δ, to the initial values $\theta_{12} = \pi/4$ and $\theta_{13} = 0$ of the mixing angles even if ϕ is large.[4]

Given that experiments indicate a large $\theta_{23} \approx \pi/4$ and disfavour a large θ_{13}, we conclude that the degenerate case generating a VO solution to the solar neutrino problem, stable against radiative corrections, must lie very close to the bimaximal mixing solution.[5]

Up to now we have concentrated on the VO solution. Our analysis immediately applies to the SAMSW and LAMSW cases as well. In such cases, however, the required level of degeneracy is not incompatible even with a splitting of $\mathcal{O}(\epsilon)$. Hence no significant restriction on the mixing parameters arises.

In conclusion, all the three favorite oscillation solutions of the solar neutrino problem (SAMSW, LAMSW or VO) are compatible with all the three possible spectra of neutrinos ('degenerate', 'pseudo-Dirac', or 'hierarchical'). The eventual identification of VO as the true solution of the solar neutrino problem would not imply a hierarchical spectrum of neutrinos. Even with a triply degenerate spectrum, the known radiative corrections are not too large if the mixing angles have certain values motivated by symmetries. In the next section we discuss how the true neutrino spectrum could be identified by conceivable experiments.

4. Will the neutrino spectrum ever be measured?

The conclusions of the previous section make even more acute the problem of the

[4]Radiative corrections to the mixing angles in presence of two degenerate neutrinos have also been studied in [12, 5, 10, 13]. The fact that, with a 'pseudo-Dirac' spectrum and appropriate correlations between the mixing angles, a cancellation of the $\mathcal{O}(\epsilon)$ corrections takes place has been also observed in [10]. Although these delicate cancellations happen in very narrow regions of the mixing angles

$$|\theta_{13}| < \frac{m^2_{\rm sun}}{4\epsilon m^2}\,, \qquad \left|\theta_{12} - \frac{\pi}{4}\right| < \frac{m^2_{\rm sun}}{4\epsilon m^2}$$

we consider this situation of physical interest because mixing parameters inside these narrow regions are motivated by symmetries [6, 7, 8].

[5]We have shown that the radiative corrections to the solar splitting vanish at leading order in λ_τ if $V = R_{23}(\theta_{23}) \cdot R_{13}(0) \cdot R_{12}(\pi/4)$. The same thing happens for a more general $V = R_{12}(\Delta\theta_{12}) \cdot R_{23}(\theta_{23}) R_{12}(\pi/4)$, since the rotation $R_{12}(\Delta\theta_{12})$ is irrelevant as long as the μ and e Yukawa couplings are neglected. It seems not unconceivable that even a V of this form might result as a consequence of an approximate symmetry, although this is not the case in the models in [7, 8]. When rewritten in the standard parametrization (1.1) V corresponds to having $\theta_{13} \neq 0$ with $\sin\theta_{13} = \tan\theta_{23} \cdot \tan(\theta_{12} - \pi/4)$, a relation equivalent to eq. (19) in [10]. The apparently different correlation presented in eq. (35) of [13] is an equivalent parametrization of the same V.

JHEP10(1999)020

possible experimental determination of the neutrino spectrum [14].

If θ_{13} is non-zero, due to matter effects, the sign of the atmospheric mass splitting might be measurable by the study of $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations in a long baseline experiment [15], using a ν beam generated by a neutrino factory. In turn this would allow to discriminate between a 'hierarchical' spectrum (where $m_3^2 \gg m_{1,2}^2$) and a 'pseudo-Dirac' spectrum (where $m_{1,2}^2 \gg m_3^2$).

The $0\nu, 2\beta$ decay searches have set a strong constraint on the modulus of the relevant element of the neutrino mass matrix

$$|M_\nu|_{ee} = \left|c_{13}(c_{12}^2 m_1 + s_{12}^2 m_2 e^{2i\varphi_2} + s_{13}^2 m_3 e^{2i\varphi_3})\right|, \tag{4.1}$$

where φ_i are arbitrary phases At the moment, taking into account the uncertainty on the nuclear matrix element, it is $|M_\nu|_{ee} < (0.2 \div 0.4)\,\mathrm{eV}$ [16]. The sensitivity of $0\nu, 2\beta$ experiments is thought to be improvable by about one order of magnitude [17]. A signal for a neutrino mass might also be obtained from studies of large scale structures in the universe, together with accurate measurements of anisotropies in the temperature of the Cosmic Background Radiation. With the standard cosmological model as reference paradigm, a sensitivity to a total neutrino mass $\sum_\nu m_\nu \gtrsim 0.3\,\mathrm{eV}$ is thought to be achievable [18]. The impact of these searches on the issue under consideration can be summarized as follows, as explained below:

1. Finding a $0\nu, 2\beta$ and/or a cosmological signal, at the level specified above, would prove the 'degenerate' or the 'pseudo-Dirac' spectrum. Different signals can discriminate between 'degenerate' or 'pseudo-Dirac' spectra and/or imply a specific solution of the solar neutrino problem.

2. If neither $0\nu, 2\beta$ or a cosmological signal will be found, further progress will require knowing that $|M_\nu|_{ee} < m_{\mathrm{atm}}$. In such a case, establishing SAMSW for solar neutrinos will prove the 'hierarchical' spectrum. On the contrary, LAMSW or VO for solar neutrinos would not allow any straightforward conclusion on the spectrum itself.

Finding $|M_\nu|_{ee} > 0.01\,\mathrm{eV}$ and/or $\sum_\nu m_\nu > 0.3\,\mathrm{eV}$ would be against the 'hierarchical' spectrum since, in such a case

$$|M_\nu|_{ee} \leq |s_{13}^2 m_{\mathrm{atm}} + m_{\mathrm{sun}}| \lesssim 0.01\,\mathrm{eV}$$

and

$$\sum_\nu m_\nu \approx m_{\mathrm{atm}} \lesssim 0.1\,\mathrm{eV}$$

upon use of (1.2), (1.3) and (4.1). Specifically, finding

$$|M_\nu|_{ee} > m_{\mathrm{atm}} \qquad \text{and/or} \qquad \sum_\nu m_\nu > 2m_{\mathrm{atm}}$$

JHEP10(1999)020

would prove the threefold degenerate spectrum. Furthermore, finding

$$0.01\,\mathrm{eV} < |M_\nu|_{ee} < m_{\mathrm{atm}}$$

and/or

$$\sum_\nu m_\nu > 3|M_\nu|_{ee}$$

would exclude the SAMSW solution of the solar neutrino problem. Finally, the only existence of a bound on $|M_\nu|_{ee}$, $|M_\nu|_{ee} < m_{\mathrm{atm}}$, together with SAMSW for solar neutrinos would prove the 'hierarchical' spectrum, since, for 'degenerate' or 'pseudo-Dirac' neutrinos, the smallness of θ_{12} implies

$$|M_\nu|_{ee} \approx \max m_\nu \geq m_{\mathrm{atm}}\,.$$

In summary, disappointing as it may be, the experimental distinction between the different neutrino spectra may be hard to achieve in absence of a $0\nu, 2\beta$ or a cosmological signal and with the solution of the solar neutrino problem proven to be either LAMSW or VO. However, the VO solution of the solar neutrino problem will be incompatible with a degenerate spectrum if supersymmetry with $\tan\beta \gtrsim 10$ will be discovered. Of course it is also possible that solar oscillations are not due to one of the three standard solutions and/or that the LSND anomaly [19] will be confirmed.

References

[1] B.T. Cleveland et al., *Nucl. Phys.* **38** *(Proc. Suppl.)* (1995) 47;
Super-Kamiokande collaboration, *Phys. Rev. Lett.* **82** (1999) 2430 [hep-ex/9812011];
K. Lande et al., *Nucl. Phys.* **77** *(Proc. Suppl.)* (1999) 13;
SAGE collaboration, *Nucl. Phys.* **77** *(Proc. Suppl.)* (1999) 20
Gallex collaboration, *Nucl. Phys.* **77** *(Proc. Suppl.)* (1999) 26;

[2] Super-Kamiokande collaboration, *Phys. Lett.* **B 436** (1998) 33 [hep-ex/9805006];
M. Messier, *Atmospheric Neutrinos with* Super-Kamiokande, talk presented at the 1999 DPF meeting, Jan 1999, available at
http://hep.bu.edu/~messier/dpf/index.html;
D. Casper, *Evidence for atmospheric neutrino oscillation*, talk presented at the XXXIV Recontres de Moriond on Electroweak interactions and unified theories, Les Arcs, March 13–20, 1999, available at
http://moriond.in2p3.fr/EW/transparencies.

[3] For recent analyses of solar data see J.N. Bahcall, P.I. Krastev and A. Yu Smirnov, *Phys. Lett.* **B 436** (1998) 243 [hep-ph/9807216];
V. Barger and K. Whisnant, *Phys. Lett.* **B 456** (1999) 54 [hep-ph/9903262];
M.B. Smy, *Solar neutrinos with* Super-Kamiokande, hep-ex/9903034;
P.I. Krastev, *Global analysis of solar neutrino data*, hep-ph/9905458, see also [6] for a non-standard analysis.

JHEP10(1999)020

[4] J. Ellis and S. Lola, *Can neutrinos be degenerate in mass?*, hep-ph/9904279.

[5] J.A. Casas, J.R. Espinosa, A. Ibarra and I. Navarro *Naturalness of nearly degenerate neutrinos*, hep-ph/9904395; *Nearly degenerate neutrinos, supersymmetry and radiative corrections*, hep-ph/9905381.

[6] R. Barbieri, L.J. Hall, D. Smith, A. Strumia and N. Weiner, *J. High Energy Phys.* **12** (1998) 017 [hep-ph/9807235].

[7] R. Barbieri, L. Hall and A. Strumia, *Phys. Lett.* **B 445** (1999) 407 [hep-ph/9808333].

[8] R. Barbieri, L.J. Hall, G.L. Kane, G.G. Ross, *Nearly degenerate neutrinos and broken flavour symmetry*, hep-ph/9901228.

[9] F. Vissani, *A study of the scenario with nearly degenerate Majorana neutrinos*, hep-ph/9708483;
H. Georgi and S. Glashow, *Neutrinos on earth and in the heavens*, hep-ph/9808293.

[10] J.A. Casas, J.R. Espinosa, A. Ibarra and I. Navarro, *J. High Energy Phys.* **09** (1999) 015 [hep-ph/9906281].

[11] Y-L. Wu, *Renormalization-group effects in the* SO(3) *gauge model with nearly degenerate neutrino masses and bi-maximal mixing*, hep-ph/9905222.

[12] N. Haba, N. Okamura and M. Sugiura, *The renormalization group analysis of the large lepton flavor mixing and the neutrino mass*, hep-ph/9810471.

[13] P. H. Chankowski, W. Królikowski and S. Pokorski, *Fixed points in the evolution of neutrino mixings*, hep-ph/9910231.

[14] For related considerations see V. Barger and K. Whisnant, *Majorana neutrino masses from neutrinoless double beta decay and cosmology*, hep-ph/9904281.

[15] E. Akhedov, P. Lipari and M. Lusignoli, *Phys. Lett.* **B 300** (1993) 128 [hep-ph/9211320].

[16] L. Baudis et al., *Phys. Rev. Lett.* **83** (1999) 41 [hep-ex/9902014].

[17] J. Hellmig, M. Hirsch and H. Klapdor-Kleingrothaus, *J. Phys.* **G 24** (1998) 483.

[18] W. Hu, D. Eisenstein and M. Regmark, *Phys. Rev. Lett.* **80** (1998) 5255 [astro-ph/9712057].

[19] LSND collaboration, *Phys. Rev.* **C 54** (1996) 2685 [nucl-ex/9605001]; *Phys. Rev.* **C 58** (1998) 2489 [nucl-ex/9706006].